Optimizing 3D Tetrahedral Grids

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Tetrahedral grids are used in a wide variety of finite element applications in which good results are dependent on the quality of the grid. The Los Alamos grid team has devised and implemented in LaGriT, the Los Alamos Grid Toolbox(1), a method called graph massage that improves the geometric quality of a tetrahedral grid while preserving the shape of material interfaces and exterior boundaries.

Graph massage uses a combination of four techniques: node merging, edge refinement, node smoothing and edge reconnection. It performs up to four iterations of a loop which first automatically merges close nodes, then reconnects edges, then smooths nodes. At the end of this loop, long edges are refined and there is a final edge reconnection step.

Graph massage improves grid quality ...and guarantees that the damage of any operation is bounded.

Graph massage accepts three numbers as input: a maximum edge length, a minimum edge length, and a damage tolerance. The first two input parameters control the node merging and edge bisection operations. Graph massage will attempt to merge the two end point nodes of edges that are shorter than the input minimum edge length; it

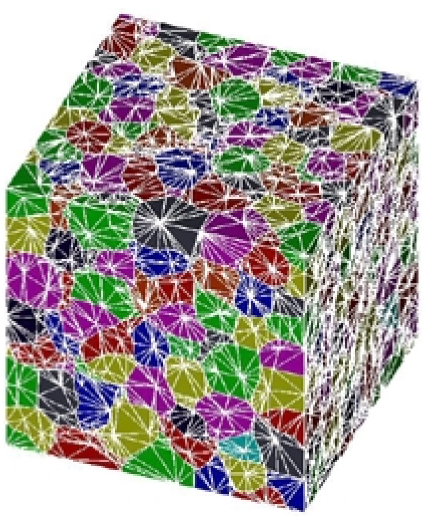

Figure 1: Original grid.

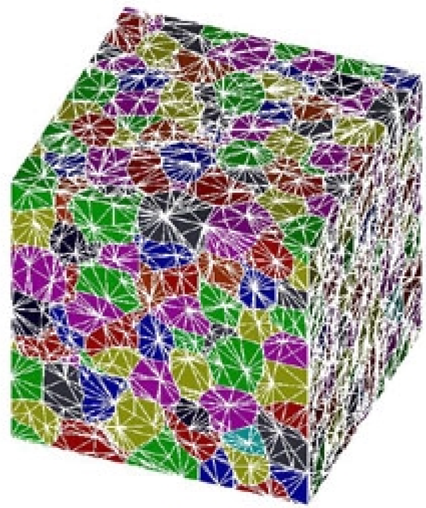

Figure 2: Grid after graph massage using a tight damage tolerance of 0.005. Grid shows very little change due to the damage tolerance that requires interface nodes not be deleted or moved.

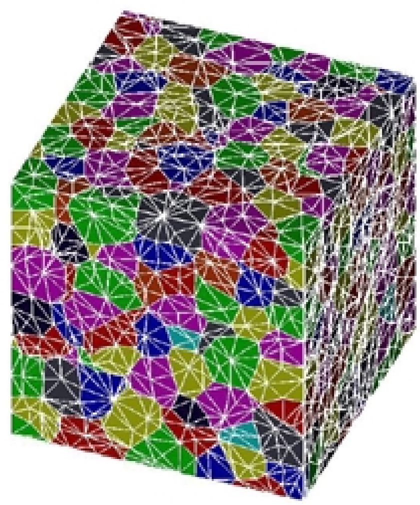

Figure 3: Grid after graph massage using a damage tolerance of 0.02 or 1% of the cube-edge length. The number of elements has been reduced by more than a factor of four and the smallest element has been enlarged by four orders of magnitude.

will bisect edges that are longer than the maximum length. The reconnection steps will attempt to create 'nice' elements by using edge swapping. For the examples presented here the meaning of 'nice' is a geometric measure that may be thought of as creating plump elements with approximately equal edge-lengths. Node smoothing will adjust node position to increase the radii of the inscribed spheres of elements containing the node.

The third input parameter, damage tolerance, controls how much the grid will be deformed by the optimization operations of merging, bisection, reconnection and smoothing. The 'damage' is a measure of how much interfaces and external boundaries are deformed. Roughly, it measures the depths of 'dents' that are invariably introduced when nodes are moved or merged and edges are swapped. This dent is the distance from the original interface or exterior surface to the new position. We guarantee that the damage of any operation is bounded by the damage tolerance. Therefore, if it is set to an extremely small number, one can expect only small node movements, few node annihilations, and few edge swaps involving elements at curved portions of material interfaces or boundaries. Conversely, if the damage tolerance is set to a large number, physical interfaces may be significantly deformed by the action of graph massage.

We demonstrate the results of running graph massage on a grid created for a grain growth simulation [2]. The original grid shown in Figure 1 consists of 500 grains in a 2 x 2 x 2 cube. This mesh has 372895 elements and the smallest element has a volume of 6.03×10^{-12}. Graph massage was then run using 0.27 as the bisection length, 0.09 as the merge length, and 0.005 as the damage tolerance. The resulting grid is shown in Figure 2. It has 349438 elements and the smallest element has a volume of 5.98×10^{-11}. Because the damage tolerance is set so low, very little change occurs. Running massage with the same bisection and merge lengths, but increasing the damage tolerance to 0.02, results in a mesh shown in Figure 3 that contains 81493 elements–approximately one quarter of the original number of elements. The smallest element has a volume of 7.77×10^{-8}, an improvement of four orders of magnitude.

[1] D. C. George, "LaGriT User's Manual," http://www.t12.lanl.gov/~lagrit.

[2] D. C. George, N. Carlson, J. T. Gammel, A. Kuprat, "3D Modeling of Metallic Grain Growth," Proceedings of Modeling and Simulation of Microsystems, San Juan, Puerto Rico, April 19-21, 1999, pp. 463-466, ISBN 0-9666135-4-6, (LA-UR-99-986, Los Alamos National Laboratory)

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